INDIAN STATISTICAL INSTITUTE, BANGALORE CENTRE
B.MATH - Second Year, Second Semester, 2021-22
Statistics - II, Midterm Examination, March 10, 2022
Answer all questions   Maximum Marks: 50   Time: $2\frac{1}{2}$ hours

1. Let $X_1, X_2, \ldots, X_m$ and $Y_1, Y_2, \ldots, Y_n$ be independent random samples, respectively, from $N(\mu, 1)$ and $N(2\mu, 1)$, where $-\infty < \mu < \infty$ is the unknown parameter.
(a) Does this model belong to the exponential family of distributions? Justify.
(b) What is minimal sufficient for μ?
(c) Find the MLE of μ.
(d) Suppose $m = n$ and let $\bar{X}$ and $\bar{Y}$ be the respective sample means. Consider the class of all unbiased estimators of μ of the form $a\bar{X} + b\bar{Y}$ where a and b are real numbers. Find the estimator in this class with the smallest variance. [4+4+6+6]

2. Let $X \sim$ Poisson(λ), $\lambda > 0$. Define $\theta = \lambda^2 \exp(-2\lambda)$.
(a) Show that $E\left\{(-1)^X X(X-1)\right\} = \theta$.
(b) Consider a random sample $X_1, X_2, \ldots, X_n$ from the X population. Find the UMVUE of θ. [5+7]

3. Let $\mathcal{G}(\lambda, \alpha)$ denote the Gamma distribution with parameters $\lambda > 0$ and $\alpha > 0$ and with the p.d.f.
$f(x|\lambda, \alpha) = \frac{\lambda^\alpha}{\Gamma(\alpha)} \exp(-\lambda x) x^{\alpha - 1}$, $x > 0$.
Suppose $X_1 \sim \mathcal{G}(\lambda, 5)$ and is independent of $X_2 \sim \mathcal{G}(\lambda, 6)$, and let $Y = X_1 + X_2$, $Z = X_1/Y$.
(a) Show that Y is complete sufficient for λ.
(b) Show that Y and Z are independently distributed. [5+4]

4. Consider X_1, X_2, X_3, X_4 i.i.d Bernoulli(p), $0 < p < 1$ and let $Y = X_1 + X_2$, $Z = X_3 + X_4$.
(a) Find the conditional distribution of (X_1, X_2, X_3, X_4) given (Y, Z) and using it show that (Y, Z) is sufficient for p.
(b) Show that (Y, Z) is not minimal sufficient for p. [5+4]